

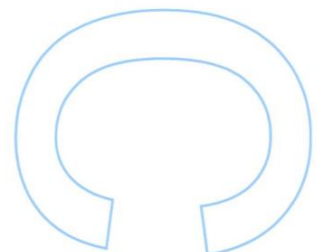
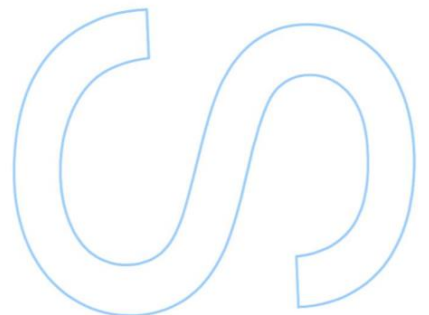
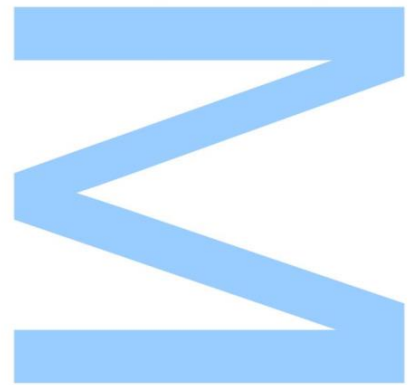
Interaction of Planetary Boundaries in the context of the Landau-Ginzburg model to understand the Earth System

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Abstract

The impact of Human Activities on the Earth System has become recently quite noticeable. The need to implement an action protocol to mitigate these effects is of great importance. The solution to this problem may lie closely to the recently proposed Planetary Quotas and Planetary Boundaries. However, once established, these Quotas may not remain valid for a long time period due to the existing interactions among the various subsystems that compose Earth System.

In this work it is presented a method to predict the validity of the Quotas for the typical timescales of the associated interactions, in particular, between atmospheric carbon dioxide's concentration and oceans' acidification. A model for these planetary boundaries is proposed considering the physical processes associated with each one.

After considering the modeling for both Planetary Boundaries, a calculation a calculation is presented. The numerical result allows for the conclusion that it is viable to implement Planetary Quotas for the typical timescale of the interactions, about an year, due to the low order of magnitude of the interaction term.

Keywords—Anthropocene; Accounting System; Climate Change; Ocean Acidification; Carbonate System; Carbon Dioxide Emissions

Resumo

O impacto das Actividades Humanas no Sistema Terrestre tem vindo a sentir-se cada vez mais fortemente. A necessidade de implementar um protocolo de ação para mitigar estes efeitos tem portanto uma importância acrescida. A solução para este problema poderá estar nas, recentemente propostas, Quotas Planetárias e nas *Planetary Boundaries*. No entanto, uma vez estabelecidas, estas Quotas poderão não se manter válidas durante muito tempo devido a existência de interações entre os vários subsistemas que compõe o Sistema Terrestre.

Neste trabalho apresentar-se-á um método para prever a validade das Quotas Planetárias durante as escalas temporais típicas das interações associadas com as mesmas. Em particular aprofundar-se-á para dois *Planetary Boundaries* específicos que se consideram ser os de maior importância e com maior impacto, concentração de dióxido de carbono na atmosfera e a acidificação dos oceanos. Uma possível modelação para estes *Planetary Boundaries* é proposta tendo por base considerações sobre processos físicos associados.

Após a modelação de ambos os *Planetary Boundaries* enunciados anteriormente, é apresentado o cálculo do termo de interação entre os mesmos. O resultado permite concluir que a implementação das Quotas Planetárias para uma escala típica de cerca de um ano é válida devido à baixa ordem de grandeza do termo de interação.

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Chapter 1

Introduction

Over the past decades, the impact of Anthropogenic activities on our planet, has reached an alarming scale. The continuous increase in the average planetary temperature, natural disasters becoming more common, accelerated loss of biodiversity are just some examples of the consequences that come from Human Activities. A new approach to global sustainability was proposed in Ref.[1] in which a set of control parameters of the Earth System was defined. These were named Planetary Boundaries. By defining these parameters it is possible to determine threshold values that when exceeded may lead to catastrophic, non-linear events throughout the system. The idea is to keep the parameters under these values, in the so-called Safe Operating Space. So far nine Planetary Boundaries have been proposed, such as climate change, ocean acidification, stratospheric ozone, biogeochemical nitrogen cycle and phosphorus cycle, global freshwater use, land system change, the rate at which biological diversity is lost, chemical pollution and atmospheric aerosol loading. From the nine Planetary Boundaries proposed two will be considered in this work, namely Climate Change and Ocean Acidification.

The main objective of this work is to model the planetary boundaries in terms of physical variables. In the case of the two highlighted planetary boundaries the variables are concentrations of the entity that best describes them, carbon dioxide concentration in the atmosphere and hydrogen ion concentration. These two planetary boundaries were chosen since they might be considered the most important ones as well as being the most, seemingly, straightforward to model due to the knowledge of the underlying physical processes.

Moreover, the interaction between these two terms is very important in regulating the Earth System and an important part of the modeling goal is to calculate these type of terms.

This approach of considering possible interaction terms provides a broader physical support to the accounting framework of Ref.[2], implying that such is useful as long as the interaction terms are less relevant than the individual terms and do not evolve significantly in a time scale in which their effect becomes relevant.

Chapter 2

A Physical framework to derivate transformations of the Earth System

2.1 Free Energy of the Earth System

The model proposed in Ref. [3] was created to capture the underlying Physics of the processes that characterize the Human Activities' impact. It was considered that the transition from the geological epoch known as the Holocene to the geological epoch we live today, the Anthropocene, is a phase transition that is induced by the existence of an external force, the Human Activity. This force overpowers the natural processes of the system and, consequently, the Holocene system's stability is disrupted. Previous transitions were driven by natural causes. In Physics it is well known how to describe these phase transitions using the Landau-Ginsburg Phase Transition Theory.

It is proposed that the Free Energy function F , equation (2.1), that characterizes the free energy of the Earth System, is governed by natural variables, q . Therefore the coefficients a and b that appear in the Free Energy Equation are functions of the natural variables q . These functions $a(q)$ and $b(q)$ should reflect the system's evolution when driven by natural causes, eg: astronomical; geological, etc. Since during the Anthropocene the major changes in the system are Human-induced, then $a(q)$ and $b(q)$ must be much smaller than the Human activity function, as explained in Ref. [3]. The order parameter for this model, ψ , is chosen to be Earth System's average temperature deviation from that of the Holocene, at a given time; $\langle\psi\rangle = \frac{T-T_H}{T_H}$ where T_H is the Holocene's temperature, so that it empha-

sizes that the phase transition occurs for deviations from this symmetry temperature. It is noteworthy that since the order parameter is written in terms of the Earth's average temperature, it follows that $\langle\psi\rangle$ is spatially homogeneous; therefore any terms in the free energy equation that are position dependent may be eliminated from it. The equation then takes the following simpler form,

$$F = F_0 + a(q)\psi^2 + b(q)\psi^4 - H\psi. \quad (2.1)$$

The Human Activity is presented as a function H in the free energy equation. This function is composed by the sum of h'_i s terms as well as a sum of interaction terms between them, [4]:

$$H = \sum_{i=1}^9 h_i + \sum_{i,j=1}^9 g_{ij}h_ih_j + \sum_{i,j,k=1}^9 \alpha_{ijk}h_ih_jh_k + \dots \quad (2.2)$$

At this point, the proposed model is of great value since it allows for the modeling of each Planetary Boundary in physical variables as well as for interaction terms of the various components that compose the Earth System. This is crucial since it is believe that setting measures to counteract the Human Impact is redundant if the system has strong interactions with itself, that is, it may be volatile and change its state rapidly, invalidating the measures previously established. By calculating these interaction terms it may be possible to understand how they behave during a typical timescale and safely propose a time interval when it is valid to act.

The model in its most simple state has already been successful in describing the Anthropocene Equation [3, 5], the so called Great Acceleration; as well as in predicting the behaviour of the temperature deviation in terms of the Human Activity, H , in fact $\langle\psi\rangle \propto H^{\frac{1}{3}}$ [3]. This means that it is possible to predict how temperature changes by estimating the H function.

2.2 Planetary Boundaries

Each term h_i is modeled as the difference in concentration, Δx_i , between the value of the control variable at a given point in time, $x_i(t)$, and its value during the Holocene, $x_{i,H}$, multiplied by a constant, α_i such that each term has units of energy. It is important to note that when $x_i(t)$ is equal to $x_{i,H}$ the system should be in its Holocene state, in other words, the corresponding term in the function H must be identically zero, hence:

$$h_i = \alpha_i \Delta x_i = \alpha_i (x_i - x_{i,H}). \quad (2.3)$$

Each variable $x_i(t)$ must reflect how Human activities impact the Earth System directly. This means that some of them might be constant in the Human activities frame, but might change due to the interaction terms. For example, when studying the Ocean Acidification term it is clear that human activity, in general, does not directly change the water's pH . However, anthropogenic carbon emissions will increase carbon dioxide's concentration in the ocean and consequently hydrogen ion concentration, leading to a pH increase.

Identifying climate change as h_1 it seems natural to consider the carbon dioxide concentration in the atmosphere as its control variable x_1 . As far as the x_1 modelling goes, it is first considered that at some time, t_0 , there is a concentration $x_1(t_0) = x_{1,0}$ such that the system is near equilibrium. Immediately after time t_0 there is an increase of the concentration due to an 'external' input, Anthropogenic emissions. For simplicity it is considered that over one year, the timescale that will be taken into account throughout this work, the emission of carbon dioxide has a constant value η , such that by the end of an year the total carbon dioxide emitted by Human activities is $\eta(t - t_0) = \epsilon$. Taking this into account it follows,

$$x_1 = x_{1,0} + \eta(t - t_0). \quad (2.4)$$

Therefore, the energy contribution to function H by carbon dioxide in the atmosphere is,

$$h_1 = \alpha_1 (x_{1,0} - x_{1,H} + \eta(t - t_0)). \quad (2.5)$$

Identifying now the oceans' acidification with h_2 , it seems natural to consider the concentration of hydrogen ion, H^+ , in seawater as the control variable, x_2 . As stated earlier,

contrary to x_1 , x_2 does not increase directly because of Human activities but due to the interaction between the oceans and the atmosphere. On the other hand, similarly to x_1 , it is considered that at some point in time the system is near equilibrium and there is an H^+ equilibrium concentration, $x_{2,0}$. This means that any variation in the concentration from a near equilibrium state should come from the interaction terms $g_{i,j}$. With this in mind it follows that the oceans' acidification term is,

$$h_2 = \alpha_2(x_{2,0} - x_{2,H}). \quad (2.6)$$

To calculate the interaction term g_{12} two different scenarios are considered, the non-interacting and the interacting ones. In the first scenario, since there is no interaction there is no change in h_2 due to carbon dioxide concentration increase, so the total variation of the H function follows,

$$\Delta H|_{annual} = \alpha_1(x_{1,0} - x_{1,H} + \epsilon^1) + \alpha_2(x_{2,0} - x_{2,H}). \quad (2.7)$$

On the other hand, in the interacting scenario, $g_{12} \neq 0$, it is known that the variation of H over the estimated time scale to achieve a quasi-static equilibrium is,

$$\begin{aligned} \Delta H|_{annual} = & \alpha_1(x_{1,0} - x_{1,H}) + \alpha_2(x_{2,0} - x_{2,H}) + \\ & + \alpha_1\delta x_1 + \alpha_2\delta x_2, \end{aligned} \quad (2.8)$$

where δx_1 and δx_2 are changes in the concentrations of carbon and hydrogen ion respectively, due to the existence of interaction. These quantities shall be calculated later. All factors considered, the proposition is that by adding the interaction term to the equation that resulted from the non-interaction scenario it should result in the equation from the interaction scenario,

$$\Delta H(g_{12} = 0) + 2g_{12}h_1h_2 = \Delta H(g_{12} \neq 0), \quad (2.9)$$

¹Recalling $\epsilon = \eta\Delta t$ for Δt equal to 1 year

$$\epsilon + 2g_{12}h_1h_2 = \alpha_1\delta x_1 + \alpha_2\delta x_2, \quad (2.10)$$

yielding an equation for the value of the interaction term that can be solved to obtain an expression for the interaction term, g_{12} , from empirical data,

$$g_{12} = \frac{\alpha_1(\delta x_1 - \epsilon) + \alpha_2\delta x_2}{2\alpha_1\alpha_2(x_{1,0} - x_{1,H} + \epsilon)(x_{2,0} - x_{2,H})}. \quad (2.11)$$

The previous equation shows that the existence of an interaction between the carbon dioxide in the atmosphere and the ocean acidity leads to perturbations, δx_i 's, of the control variable. But there is still the need to develop a method to obtain these variations as an output given the annual carbon emission values as an input. It is here that the knowledge of the carbonate system is particularly relevant. Every term in these equations is known apart from the perturbations that come from the interaction, δx_1 and δx_2 . In the following section it will be shown how to calculate these values.

Chapter 3

The Dynamics of the Oceans' Acidification

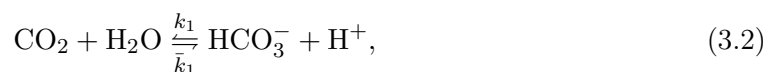
3.1 The Carbonate System

In the previous section it has been shown that the existence of an interaction between the atmosphere and the ocean leads to perturbations, δx_i 's, of the control variables. However, a method that yields these variations as outputs for the annual carbon emission input is yet to be developed. As stated earlier, it is at this point that the knowledge of the carbonate system is relevant.

The relation between the concentration of CO_2 in the atmosphere and the oceans is regulated by a chemical equilibrium, with a solubility constant K_0 ,



The chain of chemical reactions that describe the carbonate system is well known and may be broken down into two simple reactions,



where the k_i 's are the forward reaction rate and \bar{k}_i 's the reverse reaction rate coefficients.

Taking these reaction into account, it follows the discussion of the dynamics involved in the carbonate system as well as how the increase of carbon dioxide in the atmosphere due to anthropogenic carbon emissions leads to an acidification of the ocean. To do so, it is required a mathematical description of how the concentrations of all concentrations vary over time after the system's equilibrium is disturbed, that is, when there is a continuous input of carbon dioxide in the system. Recalling equations 3.2 and 3.3 as well as introducing the following notation, $v = [\text{CO}_2]$, $w = [\text{H}^+]$, $y = [\text{HCO}_3^-]$, $z = [\text{CO}_3^{2-}]$, then,

$$\frac{dv}{dt} = -k_1v + \bar{k}_1yw, \quad (3.4)$$

$$\frac{dy}{dt} = k_1v - \bar{k}_1yw - k_2y + \bar{k}_2zw, \quad (3.5)$$

$$\frac{dz}{dt} = k_2y - \bar{k}_2zw, \quad (3.6)$$

$$\frac{dw}{dt} = k_1v - \bar{k}_1yw + k_2y - \bar{k}_2zw. \quad (3.7)$$

It is important to note that, earlier, it was defined x_1 as the CO_2 concentration in the atmosphere and assigned it as the control variable for climate change. Now, v is used as the CO_2 concentration in the oceans. Since these two concentrations follow an equilibrium dictated by a solubility constant, they can be interchanged as the control variable for climate change.

Also, it is noteworthy that equation 3.4 is the time evolution of carbon dioxide's concentration simply due to the chemical dynamics, which means that it is still needed to account for the continuous input of carbon dioxide. From Ref. [6] it is obtained that over the last year (2018) the average concentration of carbon dioxide in the atmosphere has been increasing almost linearly¹, with a rate of 0.250ppm per month (from 407 ppm in January 2018 to 410 ppm in January 2019).

¹This linear approximation might be a crude one, but it is still interesting to see what yields out from simple scenario.

In Ref. [7], it is estimated that the time it takes for the exchange between gaseous and aqueous carbon dioxide to reach an equilibrium is about a year (240 days). With that in mind, our model assumes that at the start of an year (e.g January 2018) carbon dioxide exchange has reached an equilibrium and some atmospheric concentration, for this case 407 ppm, and considers a steady input of carbon such that at the start of the following year (in this scenario January 2019) the increase in concentration was of 3 ppm granting a total of 410 ppm, meaning that there was an increase of about 0.737% of CO_2 in the atmosphere. Over the year the oceans must have reached a new equilibrium and since the percentual increase was small, the new equilibrium should not be too distant from the previous one. Taking this into account, it seems reasonable to consider a Linear Stability approach to compute how the concentrations of the inorganic forms respond to a small, but steady increase of CO_2 in the system.

Defining a vector \vec{r} as:

$$\vec{r} = \begin{pmatrix} v \\ y \\ z \\ w \end{pmatrix}, \quad (3.8)$$

and a small perturbation around a fixed \vec{r} , $\vec{r} \rightarrow \vec{r} + \delta\vec{r}$. From the linear stability approach $\delta\dot{r}_i = \frac{\partial r_i}{\partial r_j}|_{\vec{r}_0} \delta r_j + \mathcal{O}(\delta r^2)$, where the dot stands for the usual time derivative and \vec{r}_0 is the fixed point for which $\delta\vec{r} = 0$. The derivative terms may be represented as components of a matrix M such that $M_{ij} = \frac{\partial r_i}{\partial r_j}|_{\vec{r}_0}$. The equilibrium condition gives the following concentrations,

$$\vec{r}_0 = \begin{pmatrix} v_0 \\ y_0 \\ z_0 \\ w_0 \end{pmatrix} = \begin{pmatrix} v_0 \\ \frac{k_1}{k_1} \frac{v_0}{w_0} \\ \frac{k_1 k_2}{k_1 k_2} \frac{v_0}{w_0^2} \\ w_0 \end{pmatrix} \quad (3.9)$$

Given these values, the matrix takes the form,

$$M = \begin{pmatrix} -k_1 & \bar{k}_1 w_0 & 0 & k_1 \frac{v_0}{w_0} \\ k_1 & -\bar{k}_1 w_0 - k_2 & \bar{k}_2 w_0 & -k_1 \frac{v_0}{w_0} + \frac{k_1 k_2}{k_2} \frac{v_0}{w_0^2} \\ 0 & k_2 & -\bar{k}_2 w_0 & -\frac{k_1 k_2}{k_2} \frac{v_0}{w_0^2} \\ k_1 & -\bar{k}_1 w_0 + k_2 & -\bar{k}_2 w_0 & -k_1 \frac{v_0}{w_0} - \frac{k_1 k_2}{k_2} \frac{v_0}{w_0^2} \end{pmatrix} \quad (3.10)$$

That is written in terms of v_0 and w_0 which can be calculated by knowing the concentration of carbon dioxide (v_0) and seawater pH (w_0) at a given time. The dynamical equations around the equilibrium point are then given by,

$$\dot{\delta \vec{r}} = M \delta \vec{r} + m \hat{e}_x, \quad (3.11)$$

where the term $m \hat{e}_x$ is the external contribution from the steady increase in the CO_2 concentration.

Chapter 4

Results

4.1 Results

Even though an approximation for the dynamics has already been considered, it is still hard to calculate the expressions for the equilibrium concentrations, therefore in this section some numerical values shall be considered and consequently the results shall be interpreted in terms of those particular values.

We start taking into account the amount of carbon dioxide that was emitted during 2017 and 2018. It is estimated that in 2017 approximately 36.1×10^{12} kg were emitted,[6], and in 2018 approximately 37.1×10^{12} kg. It is possible to convert these quantities into concentrations in the atmosphere. The conversion rate is $1 \text{ ppm}(\text{CO}_2) = 2.12 \times 10^{12} \text{ kg}$. For the purpose of forecasting, it is noteworthy that in these two consecutive years there was an increase of approximately 2.5% in carbon emission, meaning that, if this trend continues, by the end of 2019 the total amount of carbon emitted will be close to 38.0×10^{12} kg. It is helpful to summarize some relevant quantities that can be derived from the previous values.

Table 4.1: Carbon dioxide annual emission and related values. Starred values (*) are predictions based on the extrapolation of carbon emissions.

Year	2017	2018	2019
CO ₂ emissions (10 ¹² kg)	36.2	37.1	38.0*
Atmospheric CO ₂ equilibrium concentration, x_1 (ppm)	406	407	410
Oceans CO ₂ equilibrium concentration Jan. 1st, v_0 (10 ⁻⁵ mol/dm ³)	1.218	1.221	1.230
Atmospheric CO ₂ concentration increase, ϵ (ppm)	17.1	17.5	17.9*
Oceans CO ₂ concentration increase, m (10 ⁻¹⁴ mol/(dm ³ s))	1.62	1.66	1.70*

The molar concentrations in Table 4.1 can be calculated using Dalton's Law for partial pressure and the value for the solubility constant, K_0 , was estimated using the information provided in Ref. [7] assuming a temperature of 298K, which gives a value for the solubility constant $K_0 \approx 0.03 \text{ mol}/(\text{dm}^3 \text{ atm})$.

Now follows the verification of the consistency of the model and if it is possible to forecast values for the beginning of 2020. The average seawater pH in 2017 may have been very close to 8.07, Ref. [7, 8], which indicates an hydrogen ion concentration of $8.44 \times 10^{-9} \text{ mol}/\text{dm}^3$. Recalling the matrix M and the concentrations' vector, \vec{r} , described in the previous section, it is clear that in order to calculate the variations of all components' concentrations there is the need for numerical values of the equilibrium concentrations of $[\text{CO}_2]$, v_0 , and $[\text{H}^+]$, w_0 , as well as the forward and reverse reaction rate constants. The equilibrium concentrations

v_0 are given in Table 4.1 and w_0 is the value calculated from the assumed pH, namely, 8.07. The change per unit of time, m , is also shown in Table 4.1. The reaction rates are provided in [9] and are the following:

$$k_1 = 3.71 \times 10^{-2} \text{ s}^{-1}, \quad (4.1)$$

$$\bar{k}_1 = 2.67 \times 10^4 \text{ dm}^3/(\text{mol s}), \quad (4.2)$$

$$k_2 = 59.44 \text{ s}^{-1}, \quad (4.3)$$

$$\bar{k}_2 = 5.0 \times 10^{10} \text{ dm}^3/(\text{mol s}). \quad (4.4)$$

According to the definition of the Planetary Boundaries, we may identify, up to a constant, v with the atmospheric carbon concentration, x_1 , and w with the hydrogen ion concentration in the ocean x_2 . Substituting the values into Eq. (3.11) considering a time scale of one year, $t = 3.16 \times 10^7 \text{ s}$, and starting 2017 with $v_0 = 1.22 \times 10^{-5}$, $w_0 = 8.44 \times 10^{-9}$ and $m = 1.62 \times 10^{-14}$, we have that the variations in the concentrations of v and w are,

$$\delta v(2017 \rightarrow 2018) = 3.24 \times 10^{-8}, \quad (4.5)$$

$$\delta w(2017 \rightarrow 2018) = 1.84 \times 10^{-11}. \quad (4.6)$$

According to the proposed model the concentration x at the beginning of an year should be the concentration at the start of the previous year v_0 (assuming that equilibrium was achieved at the time) plus the small variation δv . So, if in 2017 there was $v_0 = 1.22 \times 10^{-5}$, then in 2018 it should have been:

$$v_0(2018) = v_0(2017) + \delta v = 1.22 \times 10^{-5}. \quad (4.7)$$

From Table 4.1, in 2018 v_0 was 1.22×10^{-5} which is fairly close to the value given in Eq. (25). Also, a new value for w_0 is calculated in a similar fashion by adding δw :

$$w_0(2018) = w_0(2017) + \delta w = 8.46 \times 10^{-9}, \quad (4.8)$$

which gives a new value for the acidity level, $pH = 8.073$. A decrease of 0.0124%.

Doing the same for the time period between 2018 and 2019, $v_0 = 1.221 \times 10^{-5}$, $w_0 = 8.461 \times 10^{-9}$ and $m = 1.66 \times 10^{-14}$:

$$\delta v(2018 \rightarrow 2019) = 3.33 \times 10^{-8}, \quad (4.9)$$

$$\delta w(2018 \rightarrow 2019) = 1.89 \times 10^{-11}. \quad (4.10)$$

From these values, the same computation as before may be performed to obtain:

$$v_0(2019) = v_0(2018) + \delta v = 1.22 \times 10^{-5}, \quad (4.11)$$

which once again is very close to the value given in Table 4.1, $v_0 = 1.23 \times 10^{-5}$ up to a 0.49% increase. The new value of w_0 is:

$$w_0(2019) = w_0(2018) + \delta w = 8.48 \times 10^{-9}, \quad (4.12)$$

which gives a new value for the acidity level, $\text{pH} = 8.072$, a decrease of 0.0124%.

So considering the values shown in Table 4.1 for the carbon emissions during 2019, and recalling that those values are assuming the same increasing trend in the amount of emissions, then it is possible to forecast values for the following year 2020. Then, for the time period between 2019 and 2020: $v_0 = 1.23 \times 10^{-5}$, $w_0 = 8.48 \times 10^{-9}$, $m = 1.70 \times 10^{-14}$:

$$\delta v(2019 \rightarrow 2020) = 3.42 \times 10^{-8}, \quad (4.13)$$

$$\delta w(2019 \rightarrow 2020) = 1.93 \times 10^{-11}. \quad (4.14)$$

Leading to the following equilibrium values for 2020:

$$v_0(2020) = v_0(2019) + \delta v = 1.23 \times 10^{-5}. \quad (4.15)$$

Assuming an 0.49% error in the previous calculation. This value of v_0 means that by the end of the year (2019) and the beginning of 2020 the carbon dioxide concentration in the atmosphere would be approximately (411 ± 2) ppm. For the hydrogen ion we get:

$$w_0(2020) = w_0(2019) + \delta w = 8.50 \times 10^{-9}, \quad (4.16)$$

which gives a new value for the acidity level, $\text{pH} = 8.07$, corresponding to a decrease of 0.0124%.

The same calculations were performed for each year of the last decade and the results are summarized in Table 4.2. From the calculation of δx_1 and δx_2 , directly obtainable from δv and δw , then Eq. (2.11) may be used to estimate the interaction term, g_{12} .

To determine the interaction term, g_{12} , recalling Eq. (2.11), for which all the values are now known, except for α_1 and α_2 . Since α_1 , α_2 and g_{12} should reflect the complex dynamics of the Earth System processes, then they should be constant. Furthermore, as the work was based only on the interaction between two processes and on a perturbative description of them, we should expect to find an approximation for the relationship between these three parameters from the data in Table 4.2.

Recalling Eq. (2.11) and using the data from Table 4.2, we can write multiple equations for g_{12} by assuming that g_{12} is constant and taking any pair of years and equating their correspondent equation, *e.g.* $g_{12}(2010) = g_{12}(2011)$. Doing this equality for all available combinations yields a sampling of experimental values for the proportionality constant α_2/α_1 , the results of which are depicted in Fig. 4.1.

With this relationship, g_{12} is then written as a function of only one of them. Because α_1 is the constant related to the greenhouse mechanism in climate change, as defined in Eq. (2.5), it seems fair to write g_{12} in terms of it up to a constant A that may be calculated from the data,

$$g_{12} = \frac{A}{\alpha_1}, \quad (4.17)$$

To each value of α_2/α_1 corresponds a values of A , which leads to the distribution depicted in Figure 4.2, with a mean value of $-1548.7 \text{ mol}^{-1}\text{dm}^3$ and a standard deviation of $307.1 \text{ mol}^{-1}\text{dm}^3$. The fact that the individual results seem to converge to a single value is reassuring that the assumptions made so far are consistent.

Table 4.2: Carbon dioxide annual emission and related values

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
CO ₂ emissions (10 ¹² kg)	33.1	34.4	35.0	35.3	35.6	35.5	35.7	36.2	37.1	38.0*	39.0*
Atmospheric CO ₂ concentration increase, ϵ (ppm)	15.6	16.2	16.5	16.7	16.8	16.8	16.8	17.1	17.5	17.9*	18.38*
Atmospheric CO ₂ equilibrium concentration, x_1 (ppm)	388	391	393	395	397	399	402	406	407	410	413*
Oceans CO ₂ equilibrium concentration, v_0 (10 ⁻⁵ mol/dm ³)	1.164	1.173	1.179	1.185	1.191	1.197	1.206	1.218	1.221	1.230	1.233*
Oceans CO ₂ concentration increase, m (10 ⁻¹⁴ mol/(dm ³ s))	1.48	1.54	1.57	1.58	1.59	1.59	1.60	1.62	1.66	1.70*	1.75*
Oceans H ⁺ equilibrium concentration, w_0 (10 ⁻⁹ mol/dm ³)	8.32	8.34	8.35	8.37	8.39	8.41	8.43	8.44	8.46	8.48	8.50*
δv (10 ⁻⁸ mol dm ⁻³)	2.89	3.02	3.08	3.12	3.15	3.15	3.18	3.24	3.33	3.42*	3.53*
δw (10 ⁻¹¹ mol dm ⁻³)	1.69	1.75	1.79	1.80	1.82	1.81	1.82	1.84	1.89	1.93*	1.99*
New oceans CO ₂ concentration, $v_0 + \delta v$ (10 ⁻⁵ mol/dm ³)	1.167	1.176	1.182	1.188	1.194	1.200	1.210	1.221	1.224	1.233*	1.240*
New oceans H ⁺ concentration, $w_0 + \delta w$ (10 ⁻⁹ mol/dm ³)	8.34	8.35	8.37	8.39	8.41	8.43	8.44	8.46	8.48	8.50*	8.52*
New atmosphere CO ₂ concentration, $x_1 + \delta x_1$ (ppm)	389	392	394	396	398	400	403	407	408	411*	415*
New oceans pH	8.079	8.078	8.077	8.076	8.075	8.074	8.074	8.073	8.072	8.071*	8.071*
Atmosphere CO ₂ concentration error	0.52%	0.25%	0.25%	0.25%	0.25%	0.50%	0.66%	~0	0.49%	-	-

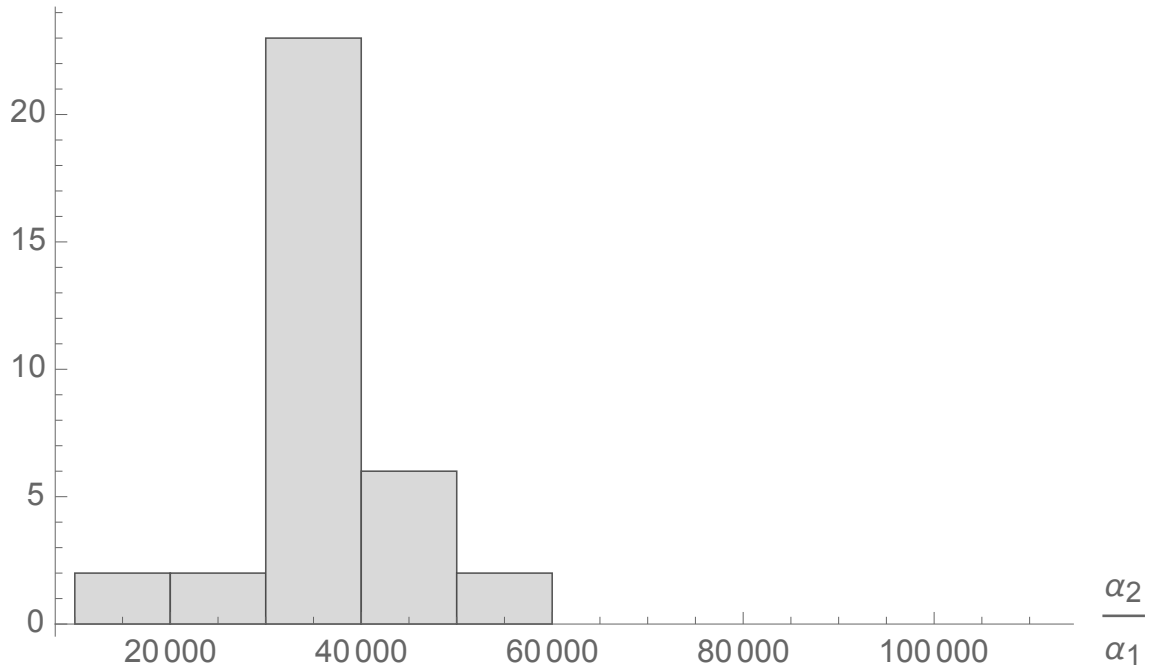


Figure 4.1: Histogram of the sample of values for the α_2/α_1 ratio from the data resulting from the years 2010 to 2018. The mean value is $\langle \alpha_2/\alpha_1 \rangle = 37131$.

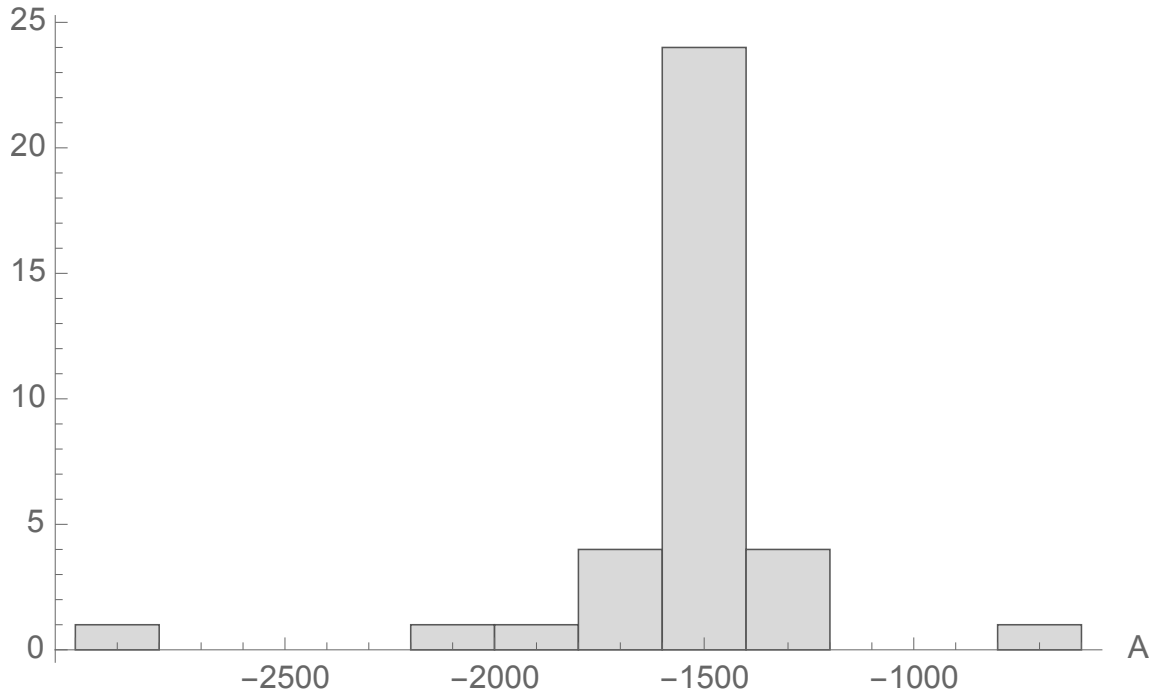


Figure 4.2: Histogram for the A constant that determines the interaction term g_{12} in terms of α_1 , as per Eq. (4.17)

It remains now to determine α_1 . For a complete determination of this parameter it would be required the knowledge of the $a(q)$ and $b(q)$ functions that control the natural drivers of the Earth System in Eq. (2.1). Still, it is possible to use the biomass accounting described in Ref. [3, 10] as a rough estimate for the purposes of getting orders of magnitude for the different terms. Thus, taking the average annual rate of biomass depletion at 10^{13} kg/year, of which around 10^{12} kg/year is living biomass, and its energy content 3.5×10^7 J/kg. The conversion of living biomass into carbon dioxide emissions is not straightforward and depends on its exact composition, but if for simplicity it is assumed that carbon atoms make up for the majority of the mass of organic matter, by comparing the molar mass of carbon and carbon dioxide, it is possible to estimate that there will be 3.6 kg of CO_2 emitted for each kg of biomass consumed, owing to the incorporation of oxygen in combustion, that matches the values in Table 4.1.

This can be further used to obtain an approximation for $\alpha_1 \approx 4 \times 10^{26}$ J/(mol dm³) that relates the increase in carbon dioxide concentration in the atmosphere with energy degradation. Using the mean value for the relationship between α_1 and α_2 as in 4.1 yields an approximation for α_2 as well, $\alpha_2 \approx 2.4 \times 10^{31}$ J/(mol dm³) and $g_{12} \approx 3.4 \times 10^{-24}$ J⁻¹. These orders of magnitude, by themselves, have little meaning, since they are comparing different things. However, the relative importance of each h_i contribution and the interaction term, g_{12} , may be compared. Thus,

$$\begin{aligned} h_1 &= \alpha_1 \Delta x_1 \approx 4 \times 10^{26} \times 5 \times 10^{-6} = \\ &= 2 \times 10^{21} \text{ J} \end{aligned} \tag{4.18}$$

$$\begin{aligned} h_2 &= \alpha_2 \Delta x_2 \approx 1.5 \times 10^{31} \times 2 \times 10^{-9} = \\ &= 3 \times 10^{22} \text{ J} \end{aligned} \tag{4.19}$$

$$\begin{aligned} g_{12} h_1 h_2 &\approx -3.4 \times 10^{-24} \times 2 \times 10^{21} \times 3 \times 10^{22} = \\ &= -2 \times 10^{20} \text{ J} \end{aligned} \tag{4.20}$$

The main takeout from these results is that the interaction term is at least one order of magnitude below the other two terms, which mean that it is relevant but should not affect the overall consistency of an accounting system, [2, 11], within the time scale of a year.

Chapter 5

Conclusion

In this work it was developed a description for two Planetary Boundaries in terms of their best suited variables, as well as a way to compute the interaction term between them. Furthermore, it was discussed the role of the interaction terms between the Planetary Boundaries parameters and how they may validate a quota system in a time scale typically closer than the slowest interacting process, [11]. A specific example was worked on, involving the interaction of the CO₂ concentration and the ocean acidity establishing the procedure to obtain a description of human action that can have interaction terms. This theoretical exercise is an important step in achieving a useful modelling description for the Earth System components that can be then inserted into its dynamical description and then establish the conditions under which it can remain within the Safe Operating Space, [1].

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